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Pair production and bremsstrahlung contributions to the stopping of relativistic heavy ions

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Abstract

We examine the energy loss to electron–positron pairs and bremsstrahlung for relativistic heavy ions penetrating matter. Pair production is the dominant source of loss at extreme projectile energies, and already at energies of a few hundred GeV/u it may add several per cent to the stopping power. An analytical formula for the pair-production contribution to the average energy loss is produced. In bremsstrahlung coherent action of the constituents of each collision partner is required in order to keep the nuclei from breaking up. This limits the emission substantially at high energies compared to the emission in collisions between pointlike non-composite but otherwise similar objects. A simple formula for the bremsstrahlung loss is presented. We conclude with remarks on the statistics of the energy-loss processes.

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1. Introduction

Electron–positron pair production plays an increasingly important role in atomic collision processes as the primary energy is raised far into the relativistic region. As an example take photoionization. For photon impact at GeV energies, the conventional channels, i.e. the photoelectric effect and Compton scattering, are outnumbered by the vacuum assisted channel in which the liberation of an atomic electron is prompted by the sparking of the vacuum, that is, by the creation of an electron–positron pair [1,2]. Similarly with the emission of radiation for charged particles: at energies beyond, typically, a few hundred MeV bremsstrahlung is the major energy-loss channel for electrons.

Electron–positron pair creation and bremsstrahlung contribute to the stopping of relativistic heavy ions penetrating matter. Since the pair-production cross section depends on the projectile

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energy $E \equiv \gamma Mc^2$ roughly as $\ln^n\gamma$ (with $n = 2$–3) [3] and pair energies typically amount to γmc^2, M and m denoting the mass of the projectile and the electron, respectively, it is obvious that the pair-production contribution to the stopping power increases at least linearly with γ. The average ionization energy loss, on the other hand, saturates into a γ-independent constant at high energies [4]. Hence, at sufficiently high energies pair production will dominate the stopping process.

Quite varied estimates of the pair-production and bremsstrahlung contributions to the stopping have appeared in the literature over the years (see [5,2] and references therein). In the present contribution we shall try to derive the correct answers.

2. Pair production

We can obtain a formula for the energy-loss rate due to pair production by application of the differential cross section provided by Racah [6]. Racah performs a full quantum calculation to lowest order of the pair production cross section for a point nucleus (charge Ze) colliding with a stationary target nucleus (charge $Z_t e$). The calculation is split into two parts according to the energy of the created pair ϵ being larger (region I) or smaller (region II) than a certain fixed energy $\bar{\epsilon}$ which fulfills the requirements $mc^2 \ll \bar{\epsilon} \ll \gamma mc^2$. Different approximations are applied in the high- and low-energy regimes. The contribution from the high-energy region I dominates the cross section, see Racah's original publication as well as [5] and the review [2] for explicit expressions. This immediately implies that for the purpose of computing the average energy loss, or even higher moments of the energy-loss distribution, we may safely disregard the low-energy region II. As discussed in the review [2], Racah's 1937 formula for the total cross section is competitive with many more recent calculations. The much simpler expression

$$\sigma = \frac{28}{27\pi} Z^2 Z_t^2 \alpha^2 r_e^2 \ln^3(\gamma/4) \tag{1}$$

aired by Heitler [3] approximates Racah's result remarkably well (α is the fine-structure constant and $r_e = e^2/mc^2$ the classical electron radius). Heitler gives another relatively simple approximate formula for the total cross section for production on a neutral atom in the limit where screening is in full action.

The analytical formula for the average energy-loss rate is obtained by integrating Racah's explicit expression for the doubly differential energy-loss cross section $\epsilon \mathrm{d}^2\sigma_\mathrm{I}/\mathrm{d}\epsilon\,\mathrm{d}w$ for region I over ϵ and the electron energy w (or rather w/ϵ). It is the lowest-order contribution in an expansion in the ratio $\bar{\epsilon}/\gamma mc^2$. The rate reads

$$-\frac{\mathrm{d}E^{\mathrm{PP}}}{\mathrm{d}x} = \pi Z^2 Z_t^2 \alpha^2 r_e^2 N \gamma mc^2 \Lambda, \tag{2}$$

where N is the density of target atoms. The lowest-order expression for the dimensionless quantity Λ is

$$\Lambda_0 = \frac{19}{9}\left(\ln\frac{\gamma}{4} - \frac{11}{6}\right). \tag{3}$$

The deviation of Λ from Λ_0 amounts to only a fraction of a per cent for $\gamma \geqslant 10^3$; for $\gamma = 10^2$ the correction is 18% (but the pair contribution to the stopping power is limited to a few per cent). Screening of the target nucleus by atomic electrons is important at high energies. It may be accounted for approximately by applying, at all energies, the following expression in place of (3):

$$\frac{19}{9}\ln\frac{183 Z_t^{-1/3}}{1 + 4e^{11/6} 183 Z_t^{-1/3}/\gamma} \equiv \Lambda_0^{\mathrm{screen}}. \tag{4}$$

Furthermore, pair production on atomic electrons may be accounted for roughly by multiplying the rate (2) by the factor $(1 + 1/Z_t)$.

As discussed in [5] and [2] the results above agree well with various results published decades back for muon stopping. Furthermore values for the energy-loss rate due to pair production obtained according to the expressions above agree within 10%, for all elements (when including the factor $(1 + 1/Z_t)$ which is important for light elements) and all energies where pair production matters, with recently tabulated muon data computed by Groom et al. [7] in a rather involved numerical scheme. As also detailed in [5,2] our results account for the moderate but systematic deviations between measured stopping powers and theoretical values for the ionization energy loss reported in [8] for bare lead ions penetrating various target materials at $\gamma = 168$.

On the basis of the formulas above and the high-energy asymptote for the electronic stopping power [4] we can produce a formula for the γ-value beyond which pair losses are higher than ionization losses [5]. The formula reads

$$\gamma = \frac{4}{Z_t \alpha^2} \frac{\ln(1.62c/R\omega_{pl})}{\Lambda_0^{screen}}. \quad (5)$$

Here R is the radius of the projectile nucleus and $\omega_{pl} = \sqrt{4\pi N Z_t e^2/m}$ the plasma frequency of the target. For Pb on Pb, a γ-value of about 1700 results by iteration (Λ_0^{screen} depends on γ).

We could have thought of applying the simpler Weizsäcker–Williams (WW) method of virtual quanta for the purpose of obtaining the energy-loss rate to electron–positron pair production; standard references to the WW method are [9,10], see also [2]. Such procedure was followed in [11]. A straight-forward application of the WW-scheme amounts to a replacement of the perturbing fields of the projectile by an equivalent bunch of photons which, in turn, interacts with the target system so as to produce electron–positron pairs according to standard expressions for photon impact (Bethe-Heitler). It gives good agreement with Racah's total cross section upon proper selection of the minimum impact parameter b_{min} which enters in the WW model. The optimum choice (half the Compton wavelength of the electron) and the level of agreement (deviations at the level of a few per cent) are discussed in [2]. However, while the cross sections come out close in the two approaches, the virtual photon method produces energy-loss rates which are lower than those obtained on the basis on Racah's cross sections typically by a factor of 2 (more if b_{min} is chosen as the Compton wavelength of the electron as in [11]). This difference can be traced to an exponential suppression of high-energy pairs, that is, pairs of energy $\epsilon > \gamma mc^2$ in the WW model. Racah's differential cross section instead shows an inverse-cube dependence on ϵ in this region. To get the high-energy tail correct in the WW model requires that pair production in the field of the projectile be included as well. That is, pair production in the rest frame of the projectile by the virtual photons of the target system has to be computed and transformed back to the laboratory system. Production at relatively large angles relative to the motion of the target system in the projectile rest frame but at basically any energy accounts for the high-energy tail missing in the first WW attempt.

3. Bremsstrahlung

In a previous publication [5] we proclaimed that earlier sources overestimated the bremsstrahlung contribution to the stopping as a result of treating the collision partners as structureless pointlike particles. Effects of finite nuclear size and requirements of coherent scattering of the constituents of both partners on each other were in general neglected. Since the length scale in bremsstrahlung emission in collisions between heavy particles is much smaller than the length scale in electron–positron pair production, we are comparing subnuclear lengths to the Compton wavelength of the electron (386 fm), such effects will be important in bremsstrahlung but not in pair creation. In [5] we did introduce a restriction based on arguments of coherent action. It did lead to substantial reduction of the bremsstrahlung yield, but not more than still allowing bremsstrahlung eventually to become the dominating energy-loss channel at extreme energies. As we shall now discuss, the restriction aired previously is not sufficient; energy losses due to bremsstrahlung are in general much smaller than those quoted in [5].

We shall apply the WW method of virtual quanta. In general there are two contributions, one from the scattering of the virtual photons of the projectile on the target, the other from the scattering of the virtual photons of the target on the projectile in the rest frame of the latter. As is apparent from Heitler's discussion of bremsstrahlung in electron–electron collisions [3], scattering on the projectile brings the major contribution. Since we discuss stopping we shall require the projectile to stay intact; processes where the projectile breaks up will be considered as separate events. It translates into a requirement of coherent action of the constituents; if the recoil in scattering were to be taken up by a single proton, this proton would in general leave the nucleus.

In the scattering of WW-photons on the projectile there are two characteristic energies: $\hbar\omega_1$ which distinguishes scattering on a single rigid object of charge Ze and mass M ($\omega < \omega_1$) from scattering on Z quasi-free protons each of mass M_p ($\omega > \omega_1$) and $\hbar\omega_2$ beyond which incoherent scattering on single protons is possible. A typical value for $\hbar\omega_1$ will be $\sim$8 MeV (binding energies per nucleon are of this order and excitation energies are typically a few MeV, giant resonances $\sim$15 MeV). The other energy is defined by the wavelength being comparable to the nuclear size R, that is, $\hbar\omega_2 \simeq \hbar c/R$ which amounts to about 25 MeV for the heaviest nuclei. The scattering is effectively classical since only photons of energies much less than the rest energy of the scatterer bring substantial contributions to the cross section (even when the scatterer is a proton).

If we reserve primed variables for the projectile rest frame the scattering cross section here reads

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega'} \sim \left\{ \begin{array}{ll} \left(\frac{Z^2e^2}{Mc^2}\right)^2 \frac{1}{2}(1+\cos^2\theta'); & \omega' < \omega_1 \\ Z^2\left(\frac{e^2}{M_\mathrm{p}c^2}\right)^2 \frac{1}{2}(1+\cos^2\theta'); & \omega_1 < \omega' < \omega_2 \\ Z^2\left(\frac{e^2}{M_\mathrm{p}c^2}\right)^2 \frac{1}{2}(1+\cos^2\theta') & \\ \quad \times\Theta\left(1-2\frac{\omega'}{\omega_2}\sin\frac{\theta'}{2}\right); & \omega_2 < \omega' \end{array} \right\}. \tag{6}$$

The last factor for the high-energy range (the Heaviside or step function Θ) restricts scattering to angles sufficiently small that the change in wave number times the size of the nucleus is small compared to 1, that is, to angles θ' smaller than approximately ω_2/ω'. This is the requirement for coherence; see [9]. To get the emitted energy per frequency bin in the laboratory the scattering cross section (6) is multiplied by the virtual photon intensity spectrum and the product is transformed to the laboratory and integrated over angles. The WW spectrum may be taken in its simplest logarithmic form since the effective cut-off at frequency $\gamma c/R_\Sigma$ appears far beyond ω_2; R_Σ is the sum of the nuclear radii of the two colliding heavy ions and enters as the minimum impact parameter. To obtain the energy loss integration over photon energy is finally performed. Very roughly, the result is

$$-\frac{\mathrm{d}E^{\mathrm{BS}}}{\mathrm{d}x} \sim \frac{16}{3}Z^2Z_\mathrm{t}^2\alpha r_\mathrm{e}^2 N2\gamma \times\left[\hbar\omega_1\left(\frac{Zm}{M}\right)^2\ln\left(\frac{\gamma c}{R_\Sigma\omega_1}\right)+\hbar\omega_2\left(\frac{m}{M_\mathrm{p}}\right)^2\ln\left(\frac{\gamma c}{R_\Sigma\omega_2}\right)\right]. \tag{7}$$

With $M \simeq 2ZM_\mathrm{p}$ for heavy ions, the mass ratios appearing inside the square brackets are comparable. Furthermore, the logarithmic factors are in lowest approximation just $\ln\gamma$.

Effectively, the bremsstrahlung intensity spectrum is constant up to an energy of $2\gamma\hbar\omega_2$ from which point it falls off fairly rapidly due to the requirement of coherence in the scattering. On the other hand, in a standard bremsstrahlung treatment (rigid pointlike objects) the spectrum extends essentially up to the primary energy. Hence, the estimate (7) is very small compared to such results; in order of magnitude the ratio of the two is $\hbar\omega_2/Mc^2$.

It is furthermore of interest to compare to the pair-production loss. If we neglect the difference in logarithmic factors, the ratio of bremsstrahlung to pair-production losses is

$$\frac{-\mathrm{d}E/\mathrm{d}x^{\mathrm{BS}}}{-\mathrm{d}E/\mathrm{d}x^{\mathrm{PP}}} \sim \left(\frac{m}{M_\mathrm{p}}\right)^2\alpha^{-1}\frac{\hbar\omega_2}{mc^2}, \tag{8}$$

that is, the ratio amounts to a few parts in a thousand. Hence, despite earlier claims, bremsstrahlung will never play a crucial role in the energy loss of relativistic heavy nuclei.

4. Statistics

The energy-loss straggling is obtained by taking the second moment of the energy-loss distribution. Since the differential cross section for pair production varies as the inverse cube of the pair energy for high losses this implies a logarithmic dependence on the maximum energy. With this relatively weak sensitivity on the maximum energy we shall still apply Racah's analytical expression for the differential cross section for region I despite some of the inherent approximations actually require $\epsilon \ll E$. Writing the average square fluctuation in energy loss acquired per unit length as

$$\frac{d\Omega^{2\,PP}}{dx} = \pi Z^2 Z_t^2 \alpha^2 r_e^2 N (\gamma mc^2)^2 X \qquad (9)$$

we obtain the following approximation to the last factor

$$X = \pi^{-2}\left[\left(\frac{11}{5}\ln^2\gamma + \frac{21}{4}\ln\gamma - 20\right)\ln\frac{M}{m} - \frac{2}{3}\ln^3\gamma + \frac{9}{2}\ln^2\gamma - 45\ln\gamma + 125\right]. \qquad (10)$$

While the dependence on γ is correct, the coefficients represent a fit. The result is effectively insensitive to the choice of $\bar{\epsilon}$. Screening is neglected.

With the expressions (2) and (9) the relative energy-loss fluctuation acquired after the penetration of a finite thickness may be expressed as

$$\frac{\Omega}{-\Delta E}\Big/\sqrt{\frac{\gamma mc^2}{-\Delta E}} = \sqrt{\frac{X}{\Lambda}} \qquad (11)$$

if losses are assumed to be due solely to pair production. A number slightly above 4 (± 10%) results upon insertion of Eqs. (3) and (10) for a wide range of mass numbers and energies ($50 \leqslant A \leqslant 250$ and $10^2 \leqslant \gamma \leqslant 10^6$). It is of interest to compare this to the pure collisional case: using the high-energy asymptotes for the straggling and the stopping power [4] the ratio defined on the left-hand side of Eq. (11) amounts roughly to $125\gamma^{-1/2}A^{-1/3}$. For $\gamma \sim 10^2$ the collisional and pair-production ratios are comparable (but losses are by far collisional). For large values of γ where pair production is the major source of energy loss the pair-creation result (11) is considerably larger than the relative straggling for the pure collisional case.

For bremsstrahlung the statistics is different since the emitted energy per energy bin is roughly constant up to the effective end-point energy E_{cut}. For the pure bremsstrahlung case the ratio defined by the left-hand side of (11) hence amounts to, approximately, $\sqrt{E_{cut}/2\gamma mc^2}$. For bremsstrahlung emitted in collisions between heavy structureless pointlike particles this is a large number ($\sim \sqrt{M/m}$). For the more realistic situation, Section 3, where bremsstrahlung actually is of minor importance the result is $\sqrt{\hbar\omega_2/mc^2}$ which leads to moderate numerical values not very different from that quoted above for pair production.

We shall close by noting that in the hypothetical case where bremsstrahlung photons are emitted at all energies up to, approximately, the primary energy, rare hard photons contribute significantly to the average energy loss. Rare hard events, however, would not be recorded in an experiment like that performed at CERN [8]. This implies that one could in principle end up in a situation where the average energy loss to bremsstrahlung was larger than the losses to pair production but yet went undetected.

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